NASA TECHNICAL REPORT

NASA TR R-259
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THE ORBITAL RECOVERY PROBLEM

PART I - AN ANALYSIS TECHNIQUE FOR RAPID DETERMINATION OF RETURN OPPORTUNITIES AND LATERAL-RANGE REQUIREMENTS FOR RECALL TO RECOVERY NETWORKS

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JUNE 1967

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SUMMARY

An analysis technique has been developed which makes possible a rapid, accurate solution of the orbital return problem in terms of the determination of recall opportunities and the lateral-range requirements associated with these opportunities for the recovery of spacecraft. The methods of application of the analysis technique through an analytical-computer solution, a computer-graphical solution, and a slide-rule solution are discussed.

The technique of analysis may be applied to the problem of return from any orbit inclination to any point or number of points on earth. In addition, the technique allows consideration of return from circular or elliptical orbits with any period.

Although the analytical-computer solution may be costly in terms of machine time, the computer-graphical solution is shown to be a practical method of obtaining rapid, economical results for a large number of constraints without adding any appreciable inaccuracy to the solution. Examples of the application of the computer-graphical solution are presented for return from circular orbits having periods of 1.5 and 10 hours and from an elliptical orbit with a period of 2 hours and apogee and perigee distances of 5165 and 3600 nautical miles, respectively. The computer-graphical technique of solution developed in part I of this report is used to select recovery sites for the wide spectrum of constraints presented separately in part II.

INTRODUCTION

Efforts to define requirements for future entry vehicles have been aided by several studies (see, for example, refs. 1 to 8) which are applicable to the problem of return from orbit. A synopsis of these studies (with the exception of ref. 8) is given in the introduction of reference 9.

The accessibility of a spacecraft to a given point on earth from a given orbit inclination and altitude in terms of lateral-range requirements has customarily been determined by lengthy graphical techniques or by an expression of the probability of recall. One such graphical technique involves the plotting of each orbital ground track of the vehicle during the day. With a known lateral-range capability, the daily recall opportunities can then be determined for any point or points on earth. Another more simplified technique (ref. 8) involves shifting a single orbital ground track along a map of the world to represent the rotation of the earth. Both techniques require the use of a planar projection of the earth. Extreme care must be taken, therefore, to retain any degree of accuracy because of the distortion in scales generally peculiar to these types of projections. These distortions usually increase with distance from the equator (latitude) and the resulting complexity of applying these techniques increases with increasing orbital inclination. Hence, although the techniques are very simple in principle, their application can be relatively difficult and/or inaccurate.

In reference 9, a single equation has been derived which allows the calculation of the shortest perpendicular great circular arc on the earth's surface (lateral-range requirement) from a given point on earth to the orbital plane of a spacecraft at any time. Single or multiple recovery sites may be considered with equal simplicity. However, because of the broad scope and the desire to maintain a generality of results, the orbital return problem was reduced in reference 9 to the two quantities of fundamental importance – lateral-range requirements and recovery-site location. It is the purpose of the present report to treat the more realistic case of orbital return with consideration of the location of the spacecraft in its orbital plane and the interaction between the motion of the spacecraft along its orbital plane and the rotation of the earth beneath the orbital plane.

In particular, it is the purpose of this paper to define a computer technique for the determination of recall opportunities and the lateral-range requirements associated with these opportunities. The emphasis is placed on a graphical simplification to this computer technique which makes possible the rapid, economical solution of the orbital return problem for any set of constraints without loss of accuracy. Several examples of the application of this technique are presented. In addition, a slide-rule-solution technique which can be applied for specific missions is presented.

The computer-graphical technique developed herein is used to select recovery sites for the wide spectrum of constraints presented in part II (ref. 10).

SYMBOLS

a	semimajor axis
b	semiminor axis
B	location of a general recovery site on earth's surface
D	drag
e	eccentricity
I	specific impulse
L	lift
n	orbit
P	semi-latus rectum, b^2/a
r	radial distance from center of earth
t	time
V	velocity
V_c	earth satellite velocity at $r = r_o$
ΔV	deorbit velocity decrement
$\overline{V} = V/V_c$	
α	inclination of orbital plane to earth equatorial plane
γ	flight-path angle
λ	latitude
λ'	lateral-range angle (latitude of recovery site or point on earth referred to orbital plane), measured in degrees of earth surface arc (1^o = 60 n. mi.)
μ	earth gravitational constant
θ	longitude

$\dot{\theta}_E$	angular rotation rate of earth
θ'	longitude of orbital plane referred to the intersection of the earth equatorial and orbital planes
$\dot{\theta}'_s$	angular rotation rate of spacecraft in its orbit
σ	retropropulsion-system structural mass fraction
τ	period

Subscripts:

a	apogee
A,B	some point on earth
d	descent trajectory
e	elliptical orbit
p	perigee
r	return opportunity
s	spacecraft
t	time of initial return opportunity
o	surface of earth
1,2,3,...	orbit number
max	maximum
req	required

ANALYSIS TECHNIQUE

General Concepts

The lateral range required for a spacecraft to return to a particular point on earth is generally measured perpendicular to the orbital plane. The approach taken in the technique of analysis developed herein for the determination of recall opportunities and the associated lateral-range requirements is to refer the latitude and longitude of a general point on the surface of the earth to the orbital plane rather than to the earth equatorial plane. The motion of the spacecraft is, of course, easily referred to its plane. This approach simplifies the solution of the orbital recovery problem and greatly reduces the effort necessary to obtain accurate results, as is demonstrated in part II (ref. 10). Thus, in order to determine the expressions from which return opportunities and requirements may be calculated, it is first necessary to obtain an expression for the longitudinal location of a general point on the earth's surface with respect to the orbital plane as a function of time. Next, the longitudinal movement of the spacecraft in its orbit must be determined. The points in time for which a simultaneous solution of the two expressions exist may be defined as the opportunities for return of the spacecraft to the recovery site. This definition of return opportunity is actually fictitious since these solutions physically represent the time at which the spacecraft crosses the point at which a great circular arc (drawn through the point on earth perpendicular to the orbital plane) intersects the orbital plane. The actual deboost time $t_{\Delta V}$ must occur at a time prior to this solution point as given by

$$t_{\Delta V} = t_{sol} - t_d$$

where t_{sol} is the time of solution and t_d is the time increment from application of deboost velocity decrement to touchdown of spacecraft. For circular orbits, t_d will be a constant for a given orbital altitude, velocity decrement, entry trajectory, et cetera. For elliptical orbits, t_d will vary, and the exact time increment will depend also upon the location of the deboost point on the orbital path. It must be stressed that the important parameter is the time of solution since this parameter defines the time increments between return opportunities. Therefore, for simplicity, in the following discussion the times of solution will be referred to as the return opportunities, with the realization that the actual deboost time will be determined by t_d.

To illustrate the importance of determining these solution points accurately, consider a vehicle in a circular orbit with an altitude of 150 nautical miles. This vehicle will move in its orbit through one revolution in 1.5 hours. During the orbital revolution of the spacecraft, however, the earth is rotating at a rate of 15 degrees per hour. Also consider the spacecraft to be in a west-east orbit so that the spacecraft movement is in the same direction as the rotation of the earth. Since the period of the spacecraft is 1.5 hours, it will complete 16 orbits per day. However, for an equatorial orbit, the period of the spacecraft referred to some longitude (point on earth) will be 1.6 hours, and 15 return opportunities will occur per day. That is,

$$\tau_B = 1.5 + \frac{22.5}{225} = 1.6$$

where

1.5 hour is the period of spacecraft for one revolution of 360° in its orbit

22.5° is the rotation of the point on the earth at some longitude θ during 1.5-hour period

225 deg/hr is the rotation rate of spacecraft relative to some point on the surface of the earth

For an equatorial orbit, then, the requirements to reach some point on the surface of the earth may be determined by assuming an initial time t for deboost during the first orbit and by determining the lateral-range requirement for times given by

$$t_r = t_{r-1} + \tau_B$$

where $\tau_B = 1.6$ for a 150-nautical-mile orbit. However, for an orbit with any inclination other than zero, τ_B will be variable and cannot be determined easily by analytical methods. The period τ_B must be determined iteratively, numerically, or graphically as will be demonstrated. Resulting calculations show that for a -30° orbit inclination and for return to a latitude of 25.5° (e.g., a site at Homestead AFB, Florida), τ_B will vary from a minimum of 1.52 hours to a maximum of 1.67 hours during a 1-day period. (If a constant value of τ_B is used, one may predict erroneously that a given vehicle can return to a particular point during some orbit.) The average value of τ_B will again be 1.6 hours and 15 return opportunities will occur daily.

Consider now a circular orbit with the same altitude (150 n. mi.), an inclination of -60°, and a return site at a latitude of 35° (e.g., Edwards AFB, California). For this case, the average value of τ_B will be 1.5 hours and 16 return opportunities will occur daily.

The number of return opportunities during a 24-hour cycle may be determined analytically for a spacecraft in a west-east orbit with $\tau_S = 1.5$ as follows:

If $|\alpha| + |\lambda| < 90^{\circ}$, 15 return opportunities will occur

If $|\alpha| + |\lambda| \geqq 90^{\circ}$, 16 return opportunities will occur

Latitude and Longitude of Point on Earth Referred to Orbital Plane

The accessibility of the spacecraft to a given ground recovery point is defined by referring the latitude and longitude of some point on earth to the orbital plane rather than to the earth equatorial plane. The equation representing the latitudinal angle between a point on the surface of the earth and the orbital plane, as developed in reference 9, is

$$\lambda' = \sin^{-1}\left[\sin \lambda \cos \alpha + \cos \lambda \sin \alpha \sin\left(\dot{\theta}_E t + \theta\right)\right] \tag{1}$$

In the application of equation (1), the following sign conventions are utilized:

(a) Northern latitudes are considered positive and southern latitudes are considered negative.

(b) West longitudes are considered positive and east longitudes are considered negative (measured from the Greenwich meridian). Therefore, the rotation rate of the earth is negative. (Note that sign convention (b) is not the same as the one used in ref. 9.)

(c) With the intersection of the orbital plane and the earth equatorial plane occurring at 0° longitude, orbits passing over the southern hemisphere at west longitudes are considered to have positive inclinations; likewise, orbits passing over the northern hemisphere at west longitudes are considered to have negative inclinations.

In defining the longitude of a point on earth referred to the orbital plane, consider figure 1. From the spherical triangle shown in figure 1(b), the following relation must hold:

$$\tan \theta'_B = \frac{\cos(\beta + \alpha)}{\cot \phi} \tag{2}$$

But

$$\cos(\beta + \alpha) = \cos \beta \cos \alpha - \sin \beta \sin \alpha \tag{3}$$

and

$$\cos \beta = \frac{\sin \theta \cos \lambda}{\sqrt{1 - \cos^2\lambda \cos^2\theta}} \tag{4}$$

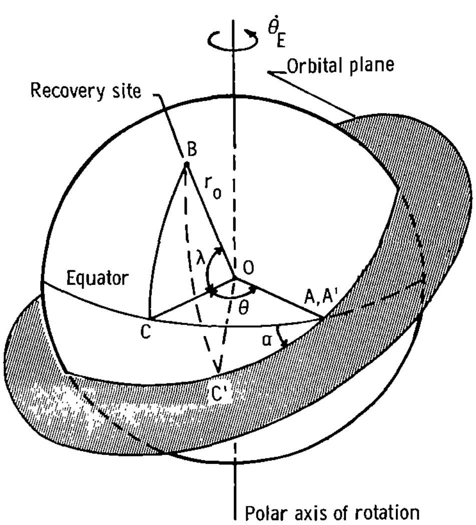
$$\sin \beta = \frac{\sin \lambda}{\sqrt{1 - \cos^2\lambda \cos^2\theta}} \tag{5}$$

Also

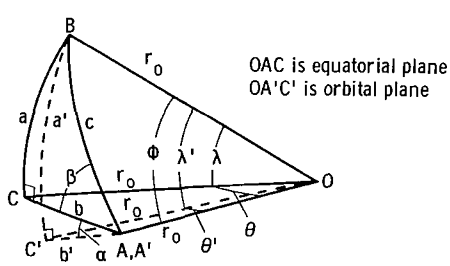
$$\cos \phi = \cos \lambda \cos \theta$$

or

$$\cot \phi = \frac{\cos \lambda \cos \theta}{\sqrt{1 - \cos^2\lambda \cos^2\theta}} \tag{6}$$



(a) Sketch of geometrical model.



(b) Sketch of spherical triangle on earth's surface.

Figure 1.- Problem geometry.

Substituting equations (3) to (6) into equation (2) yields

$$\tan \theta'_B = \frac{\sin \theta \cos \alpha - \sin \alpha \tan \lambda}{\cos \theta} \tag{7}$$

In order to include the lateral-range variation with time due to the rotation of the earth, the longitude term must include the time dependence, that is, longitude must be represented by $(\dot{\theta}_E t + \theta)$, and equation (7) becomes

$$\theta'_B = \tan^{-1}\left[\frac{\sin(\dot{\theta}_E t + \theta)\cos \alpha - \sin \alpha \tan \lambda}{\cos(\dot{\theta}_E t + \theta)}\right] \tag{8}$$

Equation (8) may be used to indicate the longitudinal motion of a given point on earth (λ, θ) referred to an orbital-plane inclination α with time. Equation (8) is necessary but not sufficient for the determination of the time variation of θ'_B since the digital computer would not be able to define the quadrant of the orbital plane in which θ'_B is measured. (Equation (8) is sufficient, however, for the application of the technique through slide-rule solution which is discussed subsequently in this report.)

The longitude θ'_B may be completely defined by deriving an alternate expression for $\sin \theta'_B$. By comparison of the signs of $\sin \theta'_B$ and $\tan \theta'_B$, the quadrant of the angle is then defined.

From figure 1, it can be seen that

$$\sin \theta'_B = \tan \lambda' \cot(\beta + \alpha) \tag{9}$$

but

$$\cot(\beta + \alpha) = \frac{1 - \tan \beta \tan \alpha}{\tan \beta + \tan \alpha} \tag{10}$$

and

$$\tan \beta = \frac{\tan \lambda}{\sin(\dot{\theta}_E t + \theta)} \tag{11}$$

Therefore, substituting equations (10) and (11) into equation (9) yields

$$\theta'_B = \sin^{-1}\left\{\tan \lambda'\left[\frac{\sin(\dot{\theta}_E t + \theta) - \tan \lambda \tan \alpha}{\tan \lambda + \sin(\dot{\theta}_E t + \theta)\tan \alpha}\right]\right\} \tag{12}$$

where λ' is given by equation (1).

SOLUTION TECHNIQUES

Analytical-Computer Solution

The lateral-range requirements necessary for a spacecraft to return from a circular orbit to a multiple-site recovery network may be calculated on a computer with the following numerical-solution technique: First, the return opportunities (i.e., time) must be determined for return to the nominal prime site within the recovery network. (See numerical example in preceding section.) This nominal prime site is defined as that site to which scheduled returns would be made under ordinary conditions. Next, opportunities for return to the remaining sites of the network can be determined with reference to those opportunities for return to the nominal prime site.

Return to prime site.- The exact determination of return opportunities requires a simultaneous solution of equations (8) and (12) for θ'_B and the equations representing θ'_s. (See appendix A.) A practical numerical approach would be as follows: Consider the vehicle to be in its orbit at a time t_1 at which the first opportunity to deboost to return to the nominal prime site occurs. The time for deboost to the nominal prime site on the second opportunity would then be given by

$$t_2 = t_1 + \tau_s + \frac{\theta'_{t_1} - \theta'_{t_1+\tau_s}}{\dot{\theta}'_s} + \frac{\theta'_{t_1+\tau_s} + \theta'_{t_1+\tau_s+t'}}{\dot{\theta}'_s} + \ldots \tag{13}$$

The general equation for time of opportunity for return to the nominal prime site may be expanded as

$$t_{r+1} = t_r + \tau_s + \frac{\theta'_{t_r} - \theta'_{t_r+\tau_s}}{\dot{\theta}'_s} + \frac{\theta'_{t_r+\tau_s} - \theta'_{t_r+\tau_s+t'}}{\dot{\theta}'_s} + \frac{\theta'_{t_r+\tau_s+t'} - \theta'_{t_r+\tau_s+t'+t''}}{\dot{\theta}'_s} + \ldots \tag{14}$$

where

$$t' = \frac{\theta'_{t_r} - \theta'_{t_r+\tau_s}}{\dot{\theta}'_s}; \quad t'' = \frac{\theta'_{t_r+\tau_s} - \theta'_{t_r+\tau_s+t'}}{\dot{\theta}'_s}; \quad \ldots$$

and the θ'_B parameters are determined from equations (8) and (12). Note that if $t_{r+1} > 24$ hours, the correct time may be obtained by subtracting 24 hours from the calculated value. The times of return opportunities are then calculated until $t_r = t_1$ which will signal the last calculation. The desired accuracy can be determined by ending the calculation of t_{r+1} when t'''' is some reasonably small number (such as, $t'''' \leqq 0.01$ hour).

The computer then stores the values of t_r for the nominal prime site and calculates the lateral range required to reach the nominal prime site by solving equation (1) by using the input latitude, longitude, orbit inclination, and times t_r.

Return to secondary sites of recovery network. - The use of the term "secondary sites" refers to all the sites within a recovery network excluding the nominal prime site. The times of opportunities for return to a secondary site are given by

$$(t_r)_B = (t_r)_A + \frac{(\theta' t_r)_A - (\theta' t_r)_B}{\dot{\theta}'_s} + \frac{(\theta' t_r)_B - (\theta' t_{r+t'})_B}{\dot{\theta}'_s} + \frac{(\theta' t_{r+t'})_B - (\theta' t_{r+t'+t''})_B}{\dot{\theta}'_s} + \ldots \tag{15}$$

where

$$t' = \frac{(\theta' t_r)_A - (\theta' t_r)_B}{\dot{\theta}'_s}; \quad t'' = \frac{(\theta' t_r)_B - (\theta' t_{r+t'})_B}{\dot{\theta}'_s}; \quad t''' = \frac{(\theta' t_{r+t'})_B - (\theta' t_{r+t'+t''})_B}{\dot{\theta}'_s}$$

until $t'^{\cdots}$ equals some arbitrarily chosen constant.

Equation (15) must be solved for each secondary site of the recovery network separately. Once the times of opportunities for return to a secondary site have been determined, the lateral-range requirements for each return opportunity may be calculated from equation (1), as was done for the nominal prime site.

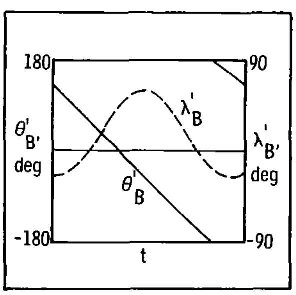
The preceding solution technique represents essentially a simultaneous solution of a sixteenth-order equation (for $\tau_s = 1.5$ hours or a low circular orbit). The computer time required for determination of return opportunities and lateral-range requirements for a large number of sites and/or orbit inclinations makes a precise numerical solution undesirable from a cost standpoint. Alternate approaches to the programing by iterative techniques in the solution of equations (8) and (12) and the equation representing the movement of the spacecraft along its orbit would also require lengthy run times for final solution. Such a solution would involve matching values of θ'_B and θ'_s through an iteration technique. For example, initially, a time t_o would be used to solve for θ'_B and θ'_s. If the solutions did not match, a new time $t_o \pm dt$ would be used until the solutions were found to converge at a time t_r. The subsequent return opportunities would be found by first calculating solutions for $t_{r+1} = t_r + \tau_s$ and then finding the simultaneous solution point by the iteration technique using $t_{r+1} = t_r + \tau_s + dt$.

A practical means of reducing costs (or keeping computer run time to a minimum) is to combine the analytical computation with a graphical solution of return opportunities as described in the following section.

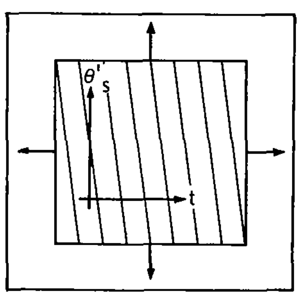
Computer-Graphical Solution

The computer-graphical technique greatly reduces machine time by determining the solution points (return opportunities) graphically. The input data required by the machine are simply the latitude and longitude of the sites within the recovery network, the orbital inclination, and the rotation rate of the earth. The computer is utilized only to determine the motion of the recovery site referred to the orbital plane. The motion of the spacecraft in its plane is determined independently. (See appendix A.)

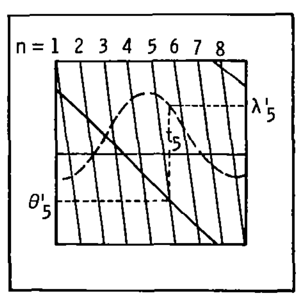
The method of solution is as follows: The computer is programed to solve equations (1), (8), and (12) for the motion of the recovery site referred to the orbital plane. Comparison of the signs of the angles θ'_B determined by equation (8) with those determined by equation (12) defines the quadrant of θ'_B. The computer output is then simply the variation of λ'_B and θ'_B with time for each site within the recovery network. The variations of λ'_B and θ'_B are then plotted as a function of time with λ'_B and θ'_B as the ordinates and time as the abscissa. A typical example of computer results is shown in figure 2. These plots may also be produced by machine with increasing accuracy as the time intervals between calculations are reduced. The movement of the spacecraft in its plane is plotted on a transparent overlay. Note that the use of an overlay allows the use of a single representation of the movement of the spacecraft in its plane with as many recovery sites as desired. The solution points (return opportunities) are then determined by placing the overlay over the plot of λ'_B and θ'_B against t for a given site (computer-graphical solution of fig. 2). Each intersection of θ'_B and θ'_s represents a return opportunity. For instance, in figure 2, the θ'_s curve of the fifth orbit intersects the θ'_B curve of the site at θ'_5 and time t_5. The required lateral range at this opportunity is determined by reading vertically from the time of the intersection to the λ'_B curve and results in a value of λ'_5. The overlay may be shifted laterally (as shown in fig. 2) to determine the best time for injection of the spacecraft into orbit at a given location for maximum return capability. For return from elliptical orbits,



Computer results



Overlay



Example of overlay used with computer plots.

Figure 2.- Computer-graphical solution.

the overlay must be shifted vertically to aline the perigee point of the orbit properly with respect to the intersection of the orbital and earth equatorial planes. This alinement is necessary since the variation of θ'_S with t is not linear for elliptical orbits. (See appendix A.) Thus, while the θ'_S variation in the overlay need extend only over 360° for circular-orbit considerations, the θ'_S variation in the overlay must extend over 540° for elliptical-orbit considerations. Examples of the application of this technique are presented in the section entitled "Examples of Application of Computer-Graphical Technique."

Slide-Rule Solution

If it is desired to determine the recall opportunities and lateral-range requirements for a particular mission with return to a few sites so that the number of calculations does not merit the computer solution of λ'_B and θ'_B as functions of time, the solution may be acquired through desk-calculator or slide-rule computation. In this case, the solution may be obtained as outlined in the section entitled "Computer-Graphical Solution." This technique may be further simplified to require only a solution of equations (1) and (8). In order to eliminate the necessity of solving equation (12), it is necessary to determine the quadrant of θ'_B at time zero and the direction in which θ'_B will vary with the rotation of the earth by analyzing a globe or map of the earth. Once this variation has been determined, the proper quadrant of θ'_B can be determined for all times in the 0 to 24 hour period. For example, if the value of θ'_B calculated for $t = 0$ is 45°, if θ'_B lies in the first quadrant, and if the orbit is west-east in direction so that θ'_B decreases with increasing time, then when $\tan \theta'_B$ changes to a negative number, θ'_B passes into the fourth quadrant.

As a check on the values of θ'_B thus calculated, if $|\alpha| + |\lambda| < 90^\circ$, θ'_B will experience a 360° variation. Conversely, if $|\alpha| + |\lambda| > 90^\circ$, θ'_B will always experience a variation of less than 180° and be in either the first and second quadrants or in the third and fourth quadrants.

EXAMPLES OF APPLICATION OF COMPUTER-GRAPHICAL TECHNIQUE

To illustrate the application of the computer-graphical technique of solution, a three-base recovery network of Homestead AFB, Florida, Edwards AFB, California, and Hickam AFB, Hawaii, will be considered for return from orbits with inclinations of -30°, -60°, and -90°. The locations of these sites are given in the following table:

Recovery site	Latitude, λ, deg	Longitude, θ, deg
Homestead AFB, Fla.	25.50	80.40
Edwards AFB, Calif.	34.90	117.85
Hickam AFB, Hawaii	21.35	157.90

In the development of this technique, the computer solution has been restricted simply to defining the movement of these sites (variation of λ'_B and θ'_B with t) relative to the orbital plane. Therefore, the computer part of the solution is independent of the orbital altitude (period) and type of orbit (circular or elliptical). The solution of the nine cases considered requires only 38 seconds of machine time. Table I lists the computer results for calculation intervals of 1.5 hours.

TABLE I.- COMPUTER RESULTS FOR CALCULATION INTERVALS OF 1.5 HOURS

t, hours	Homestead AFB, Fla.		Edwards AFB, Calif.		Hickam AFB, Hawaii	
	λ'_B, deg	θ'_B, deg	λ'_B, deg	θ'_B, deg	λ'_B, deg	θ'_B, deg
			$\alpha = -30^{\circ}$			
0	-4.14	81.3	7.64	112.7	8.05	150.6
1.5	-.54	61.3	5.00	94.4	-.67	131.5
3.0	6.40	42.2	5.95	75.9	-6.53	111.4
4.5	15.79	23.9	10.36	57.9	-8.64	90.4
6.0	26.62	5.4	17.69	40.4	-6.67	69.3
7.5	37.78	-14.7	27.21	23.3	-.93	49.2
9.0	47.79	-38.9	38.05	5.6	7.71	30.0
10.5	54.36	-69.8	49.21	-14.8	18.18	11.4
12.0	54.87	-105.2	59.10	-41.7	29.37	-8.0
13.5	49.04	-137.0	64.66	-79.7	39.96	-30.1
15.0	39.37	-162.1	62.54	-121.6	48.10	-57.1
16.5	28.27	177.3	54.22	-153.5	51.34	-89.4
18.0	17.31	158.8	43.40	-176.4	48.31	-121.8
19.5	7.64	140.5	32.26	164.9	40.30	-149.0
21.0	.28	121.5	22.00	147.7	29.77	-171.3
22.5	-3.85	101.7	13.52	130.5	18.57	169.3
24.0	-4.14	81.3	7.64	112.7	8.05	150.6
			$\alpha = -60^{\circ}$			
0	-33.74	79.6	-19.99	114.1	-6.97	150.4
1.5	-26.55	57.6	-24.90	94.8	-22.60	135.9
3.0	-13.74	40.8	-23.12	74.8	-34.12	116.0
4.5	2.34	28.3	-15.12	57.2	-38.65	90.5
6.0	20.22	18.5	-2.62	43.5	-34.42	64.9
7.5	39.10	9.9	12.70	33.2	-23.10	44.7
9.0	58.47	.5	29.70	25.5	-7.58	30.1
10.5	77.74	-18.4	47.67	20.3	10.16	18.9
12.0	80.39	-154.4	66.07	19.1	29.04	9.2
13.5	61.33	-178.9	83.33	48.8	48.45	-1.0
15.0	41.92	171.4	74.74	156.8	67.68	-17.4
16.5	22.94	162.8	56.40	161.0	81.34	-87.5
18.0	4.87	153.3	38.16	157.3	68.34	-161.7
19.5	-11.54	141.3	20.63	150.8	49.14	-178.6
21.0	-24.95	125.2	4.39	141.8	29.73	171.1
22.5	-33.14	103.9	-9.62	129.8	10.82	161.5
24.0	-33.74	79.6	-19.99	114.1	-6.97	150.4
			$\alpha = -90^{\circ}$			
0	-62.87	70.7	-46.48	123.8	-20.51	157.1
1.5	-49.87	41.9	-54.74	97.6	-40.84	151.2
3.0	-31.52	30.3	-51.60	67.1	-59.09	134.9
4.5	-11.62	26.1	-39.16	47.6	-68.65	91.0
6.0	8.66	25.8	-22.53	38.3	-59.65	46.1
7.5	28.66	29.4	-4.39	35.0	-41.54	29.1
9.0	47.37	39.5	14.00	36.1	-21.25	23.0
10.5	61.62	64.9	31.56	42.2	-.37	21.4
12.0	62.87	109.3	46.48	56.2	20.51	22.9
13.5	49.87	138.1	54.74	82.4	40.84	28.8
15.0	31.52	149.7	51.60	112.9	59.09	45.1
16.5	11.62	153.9	39.16	132.4	68.65	89.0
18.0	-8.66	154.2	22.53	141.7	59.65	133.9
19.5	-28.66	150.6	4.39	145.0	41.54	150.9
21.0	-47.37	140.5	-14.00	143.9	21.25	157.0
22.5	-61.62	115.1	-31.56	137.8	.37	158.6
24.0	-62.87	70.7	-46.48	123.8	-20.51	157.1

Circular Orbits

Low-altitude circular orbits.- Consider first a circular orbit at an altitude of 150 nautical miles having a period of 1.5 hours. The figures in appendix B show the variation of λ'_B and θ'_B with time for the three chosen sites referred to orbital inclinations of -30°, -60°, and -90°, respectively. (These results are from table I.) Overlay 1 in the back envelope represents the θ'_s movement of the spacecraft. The vehicle is considered to be initially oriented in its orbit arbitrarily at $t = 0$ over the earth reference longitude of $\theta = 0^\circ$. Therefore, aline the overlay with $\theta'_s = 0^\circ$ of the overlay corresponding to $\theta' = 0^\circ$ of the figure and with $\theta'_s = 0^\circ$ of the first orbit (on the left side of the overlay) located at $t = 0$. It should be stressed that the initial orientation selected for the nominal prime site within the network must be used for all sites within a given recovery network.

Each orbit, represented by the variation of θ'_s of 180° to -180°, is labeled on the upper and lower portion of each overlay. The intersection of a θ'_s curve with a θ'_B curve for a given orbit represents the opportunity for return to the particular site for that orbit with the time of the opportunity being the time of the intersection. The lateral-range requirement for return at this opportunity is obtained by reading vertically from the intersection point to the λ'_B curve. The times and lateral-range requirements for every return opportunity for the nine cases considered are presented in table II.

The information in table II defines the capability for recall of a spacecraft with a given lateral-range capability to the recovery network being considered. The recall capability of a spacecraft is completely independent of the method used to obtain the given lateral-range capability. Thus, the requirements are equally applicable to spacecraft which achieve lateral range by either aerodynamic maneuvering, propulsion, or a combination of the two.

To illustrate the application of the results, consider a lifting-body vehicle with a maximum hypersonic lift-drag ratio of 1.25. A vehicle with a lift-drag ratio of 1.25 should be capable of achieving a lateral range of 17° of earth surface arc. (See appendix B of ref. 9.) The recall capabilities of this lifting-body reentry vehicle are shown in table III where an $\times$ indicates the orbits in which the spacecraft can reach the particular recovery site.

From table III, it can be seen that having a recovery site at Edwards AFB, California, does not increase the opportunities of this lifting-body vehicle for return from an orbit inclined -30°. For this orbit, this lifting-body entry vehicle would be capable of return to Homestead AFB and/or Hickam AFB for ten consecutive orbits and would not be capable of return for six consecutive orbits daily. For the -60° and -90° orbit inclinations, each of the three bases is necessary for maximum capability for recall to the recovery network. For the -60° orbit inclination, this lifting-body entry vehicle would be capable of return for six consecutive orbits, incapable of return for four consecutive orbits, capable of return for the next five orbits, and incapable of return during the next orbit. Finally, for the -90° orbit inclination, this lifting-body entry vehicle would be capable of return for five consecutive orbits, incapable of return for the next four consecutive orbits, capable of return for the next five consecutive orbits, and incapable of return for the next two orbits.

TABLE II.- RETURN OPPORTUNITIES AND LATERAL-RANGE REQUIREMENTS

Orbit, n	Homestead AFB, Fla.		Edwards AFB, Calif.		Hickam AFB, Hawaii	
	t_r, hours	λ'_{req}, deg	t_r, hours	λ'_{req}, deg	t_r, hours	λ'_{req}, deg
			$\alpha = -30^{\circ}$			
1	23.62	-4.5	23.53	9.0	23.35	12.3
2	1.21	-1.2	1.06	5.3	.90	2.2
3	2.80	5.3	2.66	5.1	2.52	-4.9
4	4.38	15.0	4.22	9.5	4.08	-8.4
5	5.99	26.5	5.83	16.8	5.70	-7.2
6	7.58	38.5	7.38	26.5	7.23	-1.8
7	9.18	48.8	8.98	38.0	8.83	7.0
8	10.80	55.0	10.57	49.8	10.42	17.9
9	12.47	53.4	12.18	60.0	12.02	30.0
10	14.10	45.6	13.82	65.0	13.63	40.8
11	15.68	34.6	15.58	59.8	15.24	49.2
12	----	----	17.20	49.4	16.95	51.0
13	17.26	22.7	----	----	18.55	46.2
14	18.84	11.8	18.78	37.5	20.19	35.8
15	20.42	3.1	20.40	26.2	----	----
16	22.02	-2.7	21.95	16.5	21.78	23.9
			$\alpha = -60^{\circ}$			
1	23.60	-34.2	23.50	-17.4	23.36	0.3
2	1.22	-28.1	1.09	-24.1	.95	-17.2
3	2.80	-15.7	2.68	-24.2	2.50	-31.5
4	4.38	1.1	4.24	-16.9	4.11	-38.7
5	5.92	19.0	5.80	-4.2	5.72	-35.5
6	7.42	38.6	7.38	11.5	7.32	-25.1
7	9.00	58.5	8.92	28.9	8.85	-9.0
8	10.61	79.0	10.42	46.9	10.40	9.1
9	12.73	70.9	11.95	65.1	11.97	28.1
10	----	----	13.35	82.0	13.53	48.8
11	14.21	51.8	14.55	79.9	15.06	68.2
12	15.78	31.9	15.82	64.2	17.00	79.6
13	17.32	13.0	17.38	46.0	18.73	58.7
14	18.84	-4.8	18.83	28.4	----	----
15	20.41	-20.0	20.40	10.8	20.29	38.1
16	22.00	-30.9	21.98	-5.0	21.80	19.0
			$\alpha = -90^{\circ}$			
1	23.62	-64.2	23.43	-42.0	23.38	-11.7
2	1.34	-51.5	1.05	-53.2	.80	-31.5
3	2.91	-32.9	2.74	-53.0	2.40	-52.4
4	4.40	-12.6	4.28	-41.5	4.01	-67.7
5	5.94	7.8	5.82	-23.9	5.81	-61.2
6	7.40	27.5	7.39	-5.5	7.40	-42.9
7	8.82	45.8	8.82	12.0	8.89	-22.1
8	10.26	60.0	10.36	30.0	10.40	-1.4
9	11.62	64.0	11.80	44.8	11.95	20.0
10	12.96	55.0	13.20	54.0	13.38	39.0
11	14.40	38.7	14.59	53.6	14.80	57.1
12	15.83	20.0	15.98	44.3	16.20	68.0
13	17.37	0	17.41	29.2	17.50	64.5
14	18.84	-20.2	18.88	11.2	18.89	48.5
15	20.42	-40.7	20.39	-6.2	20.40	29.6
16	21.95	-57.0	21.92	-24.9	21.82	10.0

It is often desirable to consider return to earth in terms of the time intervals between return opportunities rather than the orbit in which return can be accomplished, particularly from the viewpoint of the command pilot of the spacecraft. This information is of importance because, even though a vehicle may be capable of return in two

TABLE III.- RECALL OPPORTUNITIES FOR LIFTING-BODY VEHICLE

Recovery site	Orbit, n															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\alpha = -30^{\circ}$																
Homestead AFB, Fla.	×	×	×	×										×	×	×
Edwards AFB, Calif.	×	×	×	×	×											×
Hickam AFB, Hawaii	×	×	×	×	×	×	×									
$\alpha = -60^{\circ}$																
Homestead AFB, Fla.			×	×									×	×		
Edwards AFB, Calif.				×	×	×									×	×
Hickam AFB, Hawaii	×						×	×								
$\alpha = -90^{\circ}$																
Homestead AFB, Fla.				×	×								×			
Edwards AFB, Calif.						×	×							×	×	
Hickam AFB, Hawaii	×							×								×

consecutive orbits, the time interval between return opportunities may approach 2τ if the first return opportunity occurs near the beginning of the orbit and the second opportunity occurs near the end of the orbit. The technique utilized herein also gives the results in this form. (See table II.) These results are plotted in figure 3 for the lifting-body vehicle with $L/D = 1.25$. Analysis of figure 3 shows that this situation does not occur for return of a lifting-body spacecraft to the recovery network considered for orbit inclinations of -30°, -60°, and -90°. That is, the available return opportunities are about equally spaced with time intervals essentially equal to the period of the spacecraft.

The effects of the selection of the initial orientation of the spacecraft in orbit relative to the location of the recovery sites can be easily illustrated by an additional example. The results in table II were obtained by assuming that the spacecraft was initially oriented in its orbit at $t = 0$ over the earth reference longitude of $\theta = 0^{\circ}$ (note that $\theta'_{s} = 0^{\circ}$ at $t = 0$). With this assumption, it was found that a lifting-body vehicle with 17° of earth-surface-arc lateral-range capability would be able to return to Homestead AFB, Florida,

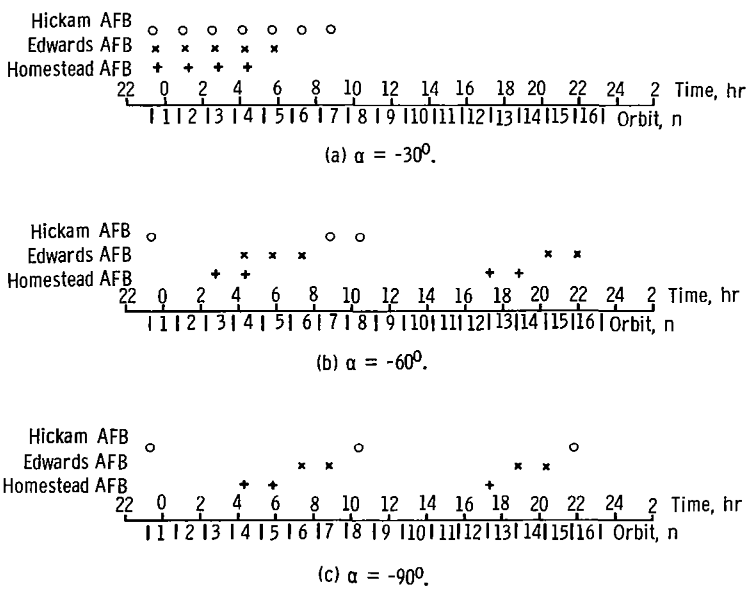


Figure 3.- Time spacing of opportunities for return of lifting-body spacecraft to sample recovery network.

TABLE IV.- RETURN OPPORTUNITIES AND LATERAL-RANGE REQUIREMENTS FOR RETURN TO HOMESTEAD AFB, FLA., WITH $\alpha = -90^{\circ}$

Orbit, n	Homestead AFB, Fla.	
	t_r, hours	λ'_{req}, deg
1	0.50	-59.8
2	2.11	-42.9
3	3.64	-22.9
4	5.17	-2.0
5	6.60	17.0
6	8.10	36.7
7	9.60	53.8
8	10.95	63.4
9	12.06	61.1
10	13.66	48.1
11	15.15	29.8
12	16.60	10.3
13	18.11	-10.0
14	19.62	-30.1
15	21.15	-49.5
16	22.80	-63.1

during three orbits daily from a polar orbit. If the same vehicle in a polar orbit with a return site at Homestead AFB, Florida, and an initial orientation point of $\theta'_s = 180^{\circ}$ at $t = 0$ is considered, the times of return opportunities and the lateral range required at each opportunity are shown in table IV. The results in table IV indicate that this lifting-body vehicle would be capable of return to Homestead AFB, Florida, during four orbits daily from a -90° (polar) orbit with the proper selection of the initial orientation of the vehicle in its orbit. This example illustrates the importance of using an overlay representing the movement of the spacecraft in its orbit to determine the best initial orientation.

High-altitude circular orbits.- The application of this technique of analysis to high-altitude circular orbits is as simple as the application to low-altitude circular orbits. To illustrate, consider a vehicle in a circular orbit at an altitude of 12 730 nautical miles with the resulting orbital period of 10 hours. It is desirable to determine the recall opportunities and the lateral-range requirements for each opportunity available prior to a repetition of the first opportunity. For the previous example in which $\tau = 1.5$ hours, this repetition occurred every 24 hours. That is, the return opportunities and lateral-range requirements were cyclic daily. For an orbit with a period of 10 hours, the return opportunities and lateral-range requirements will be cyclic every 5 days. During the cycle the vehicle will complete 12 orbits. The return opportunities and lateral-range requirements are determined for a full 5-day cycle for return to Homestead AFB, Florida, from a -60° orbit inclination from figure B2(a) with overlay 2 and are shown in table V. The vehicle is considered to be initially oriented in its orbit at $\theta'_s = 180^{\circ}$ at $t = 0$.

TABLE V.- RETURN OPPORTUNITIES AND LATERAL-RANGE REQUIREMENTS FOR RETURN TO HOMESTEAD AFB, FLA., WITH $\alpha = -60^{\circ}$, $\tau_s = 10$ HOURS

Orbit, n	Return opportunity		λ'_{req}, deg
	Day	t_r, hours	
1	1	4.16	-1.5
2	---	----	----
3	1	21.85	-30.4
4	---	----	----
5	2	16.44	23.0
6	3	6.54	26.8
7	---	----	----
8	4	1.36	-27.5
9	---	----	----
10	4	18.95	-5.9
11	5	9.03	59.0
11	5	11.75	83.4
12	5	14.07	54.0

The problems inherent with large-period (high-altitude) orbits are apparent in the comparison of table V with the results given in table II for return to Homestead AFB, Florida. In table II (for $\alpha = -60^{\circ}$), return opportunities occurred for 15 of the 16 vehicle orbits, and the lifting-body vehicle with a lateral-range capability of 17° of earth surface arc was found to be capable of return to Homestead AFB, Florida, during four of these 15 opportunities. In table V with $\tau_s = 10$ hours, return opportunities occur for only eight of the 12 orbits (however, since two opportunities occur during the eleventh orbit, a total of nine opportunities results); a vehicle with a lateral-range capability of 17° of earth surface arc would be capable of return to Homestead AFB, Florida, during two of these 12 orbits or twice in 5 days.

Elliptical Orbits

The application of the present analysis technique to elliptical orbits is no more difficult than to circular orbits once the extra-atmospheric motion of the spacecraft has been defined. (See appendix A.) Consider figure B1 for the three chosen sites and an orbital inclination of -30°. The motion of the spacecraft in its orbit is given on overlay 3 as obtained from figure A2. It should be emphasized that for elliptical orbits the total variation in injection conditions requires that the overlay be moved vertically as well as horizontally to allow for longitudinal variations in the location of the perigee of the initial elliptical orbit. Thus, the overlay must cover 540° instead of the 360° used for circular orbits.

As before, the intersection of a θ'_s curve with a θ'_B curve for a given orbit represents the return opportunity at the time indicated by the intersection point. As an example, take the initial entry point as perigee deorbit with $t = 0$ and $\theta'_s = 0^{\circ}$. The times and lateral-range requirements for every return opportunity are shown in table VI. For a vehicle with a given lateral-range capability, the return opportunities are easily obtained from this table.

TABLE VI.- RETURN OPPORTUNITIES AND LATERAL-RANGE REQUIREMENTS FOR RETURN FROM AN ELLIPTICAL ORBIT, $\alpha = -30^{\circ}$, $\tau_s = 2$ HOURS

Orbit, n	Homestead AFB, Fla.		Edwards AFB, Calif.		Hickam AFB, Hawaii	
	t_r, hours	λ'_{req}, deg	t_r, hours	λ'_{req}, deg	t_r, hours	λ'_{req}, deg
1	23.70	-4.5	23.56	9.0	23.40	12.1
2	1.74	.4	1.60	5.0	1.53	-.7
3	3.90	11.7	3.73	7.8	3.60	-7.8
4	6.00	26.6	5.84	17.0	5.74	-7.1
5	8.13	42.4	7.94	30.6	7.80	.6
6	10.26	53.8	10.04	45.8	9.94	14.3
7	12.56	53.3	12.20	60.1	12.05	29.9
8	15.15	38.0	14.40	64.5	14.17	44.2
9	----	---	17.13	50.0	16.36	51.4
10	17.37	21.9	----	---	18.80	44.6
11	19.50	7.7	19.35	33.5	21.20	28.3
12	21.60	-1.7	21.50	19.0	----	---

For instance, again consider a lifting-body class of vehicle having a lateral-range capability of 17° of earth surface arc. The opportunities for return of this vehicle to the three bases considered are presented in table VII.

TABLE VII.- RECALL OPPORTUNITIES FROM AN ELLIPTICAL ORBIT FOR LIFTING-BODY ENTRY VEHICLE, $\alpha = -30^{\circ}$, $\tau_s = 2$ HOURS

Recovery site	Orbit, n											
	1	2	3	4	5	6	7	8	9	10	11	12
Homestead AFB, Fla.	×	×	×								×	×
Edwards AFB, Calif.	×	×	×	×								
Hickam AFB, Hawaii	×	×	×	×	×	×						

As was the case for the circular orbit, the inclusion of Edwards AFB, California, as a recall site does not increase the recall capability of the spacecraft. Homestead AFB, Florida, and Hickam AFB, Hawaii, allow return on eight consecutive orbits with no return opportunity during four consecutive orbits. The associated time intervals between return opportunities may be easily obtained from table VI.

It should be emphasized that both the initial spacecraft location and longitudinal perigee location of the elliptical orbit are critical to the recovery problem for return from elliptical orbits. The use of an overlay, which permits these parameters to be changed at will, allows the definition of the recall capability of a spacecraft with one figure representing the motion of the spacecraft in its orbit rather than the large number required to consider a variety of initial points and perigee locations.

CONCLUDING REMARKS

An analysis technique has been developed which makes possible the rapid and accurate solution of several phases of the problem of return from orbit – the opportunities for recall of entry vehicles to multiple landing sites, the lateral-range requirements associated with any recall-capability constraint for return to any given number of recovery sites, and the determination of a network with a minimum number of recovery sites for any set of orbital or entry-vehicle constraints. This technique may be applied to either circular or elliptical orbits with any period and inclination for return to any point or points on earth.

Of the three solution methods presented, the computer-graphical solution appears to be the most desirable in terms of economy and speed.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., November 17, 1966,
789-30-01-02-23.

APPENDIX A

DEFINITION OF SPACECRAFT MOTION FROM ORBIT TO ENTRY

Circular Orbit

The trajectory from a circular orbit to the atmosphere is, of course, independent of the location of the spacecraft in its orbit for a nominal, Hohmann type of descent. For the present analysis, the Hohmann type of descent is considered and no variation in the extra-atmospheric longitudinal range is allowed. The relation between time and the location of the entry vehicle in its orbit is therefore a linear variation and may be written as

$$\theta'_s = \theta'_o - 360 \frac{t}{\tau_s}$$

where $-180^\circ \leqq \theta'_s \leqq 180^\circ$ and θ'_o defines the initial longitudinal location of the vehicle at the injection time, $t = 0$.

Elliptical Orbit

The trajectory of the entry vehicle from the deorbit point to the entry point must be defined for the consideration of recovery from elliptical orbits since the time from deorbit to entry for the nominal descent will vary with the initial position of the entry vehicle in its orbit.

Two factors must be considered – the position of the spacecraft in its orbit and the ΔV capability of the spacecraft to initiate the descent trajectory. For descent from orbits with a small eccentricity, the deorbit ΔV requirement for a Hohmann type of transfer will generally be well within the capabilities of the spacecraft retropropulsion system. However, this ΔV requirement rapidly increases with increasing eccentricity and may require more than 50 percent of the vehicle mass to be allocated to the retropropulsion system for only moderate eccentricities. The choice of a 180° Hohmann type transfer (apogee at deorbit and perigee at entry for the descent trajectory) between the deorbit point and the ballistic perigee of the descent trajectory is arbitrary but was selected since this is the nominal descent trajectory for return from a circular orbit. The method used in the present analysis is to select, for typical retropropulsion-system parameters, a maximum retropropulsion ΔV capability which must be equal to or greater than that required to insure a Hohmann type of descent from perigee. For those portions of the orbit for which a 180° transfer is not possible, the descent trajectory necessary to reach the desired ballistic perigee is followed.

APPENDIX A

The longitudinal angles and times associated with descent from the two orbital regions may be defined with the aid of figure A1, and the general Keplerian equations of reference 11. If the ΔV required to initiate a Hohmann type of descent is less than the allowable ΔV, the following procedure is used:

$$\Delta\gamma = \gamma_e \quad \text{since} \quad \gamma_d = 0 \quad \text{for a } 180^{\circ} \text{ transfer}$$

$\overline{V}_e$ is described by the initial orbit

and

$\overline{V}_d$ is obtained from the vis-viva integral

$$\overline{V}_d = \sqrt{\frac{2r_o}{r_{a,d}} - \frac{r_o}{a_d}} \tag{A1}$$

where $a_d = \frac{r_{a,d} + r_{p,d}}{2}$, $r_{a,d} = r_e$, and $r_{p,d}$ is specified by the available entry corridor. The velocity decrement $\Delta\overline{V}$ may be obtained from the following equation:

$$\Delta\overline{V}^2 = \overline{V}_e^{\,2} + \overline{V}_d^{\,2} - 2\overline{V}_e\overline{V}_d \cos \Delta\gamma \tag{A2}$$

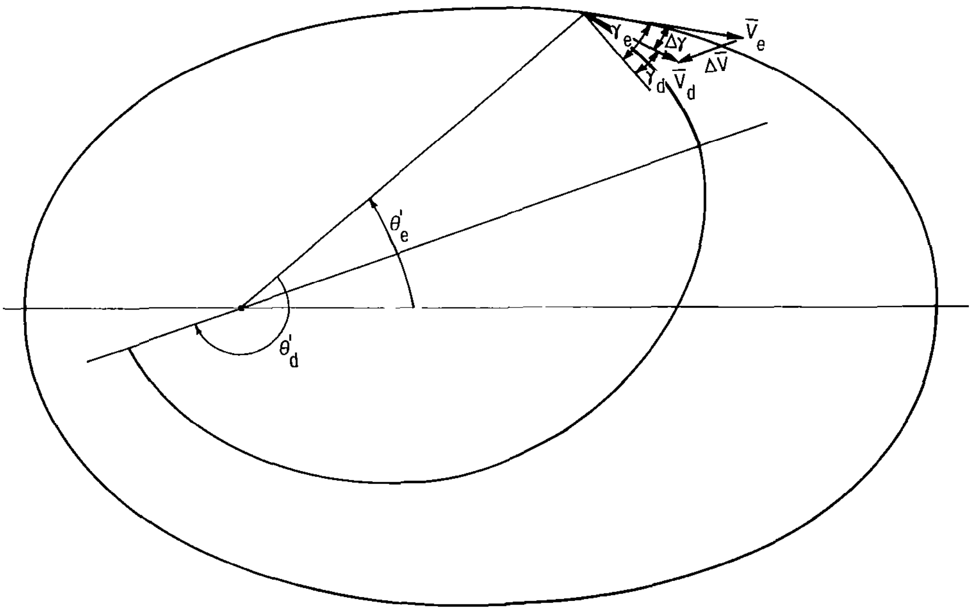


Figure A1.- Schematic drawing of descent from an elliptical orbit.

and the time elapsed from deorbit to entry, from the following equation:

$$t_d = \pi\sqrt{\frac{(r_e + r_{p,d})^3}{\mu}} \tag{A3}$$

The total time elapsed, measured from the apogee of the initial orbit, is

$$t_s = t_e + t_d \tag{A4}$$

where

$$t_e = \frac{\sqrt{\frac{r_{p,e}^3(1 + e_e)}{\mu}}}{1 - e_e}\left[\frac{\sin^{-1}\left(\frac{e_e + \cos\theta'_e}{1 + e_e\cos\theta'_e}\right) - \frac{\pi}{2}}{\sqrt{1 - e_e^2}} - \frac{e_e\sin\theta'_e}{1 + e_e\cos\theta'_e}\right] \tag{A5}$$

The location of the entry point with respect to the apogee of the initial orbit is

$$\theta'_s = \theta'_e - 180^\circ \tag{A6}$$

If the ΔV required to initiate a Hohmann type of descent is greater than the allowable ΔV, the following procedure is used: Values of $\Delta\gamma$ less than γ_e are selected and values of $\overline{V}_d$, a_d, r_d, and $r_{p,d}$ are computed from

$$\overline{V}_d = \overline{V}_e \cos\Delta\gamma - \sqrt{\overline{V}_e^2\cos^2\Delta\gamma - \left(\overline{V}_e^2 - \Delta\overline{V}_{max}^2\right)} \tag{A7}$$

$$a_d = \frac{r_o}{\frac{2r_o}{r_d} - \overline{V}_d^2} \tag{A8}$$

$$\gamma_d = \gamma_e - \Delta\gamma \tag{A9}$$

and

$$r_{p,d} = a_d\left(1 - \sqrt{1 - \frac{r_d^2}{a_d r_o}\overline{V}_d^2\cos^2\gamma_d}\right) \tag{A10}$$

The values of $r_{p,d}$ thus obtained are plotted against $\Delta\gamma$ and the desired value of $\Delta\gamma$ is obtained for the specified value of $r_{p,d}$ (obtained from entry-corridor considerations). Either an iterative or graphical solution for $\Delta\gamma$ is required because of the complexity of a direct solution involving $r_{p,d}$. The correct values of $\overline{V}_d$ and a_d, as well as the other pertinent parameters of the descent trajectory, may then be calculated from the following equations:

$$\left.\begin{aligned}
P_d &= \frac{r_d{}^2 \overline{V}_d{}^2 \cos^2\gamma_d}{r_o} \\
b_d &= \sqrt{P_d a_d} \\
e_d &= \sqrt{1 - \frac{b_d{}^2}{a_d{}^2}} \\
\cos\theta'_d &= \frac{\frac{P_d}{r_d} - 1}{e_d}
\end{aligned}\right\} \tag{A11}$$

Obviously, two values of θ'_s are obtained: One as the spacecraft is moving from apogee to perigee, the other as the spacecraft is moving from perigee to apogee. The time from deorbit to entry is obtained from the following equation:

$$t_d = \frac{\sqrt{\frac{r_{p,d}{}^3(1 + e_d)}{\mu}}}{1 - e_d} \left[\frac{\sin^{-1}\left(\frac{e_d + \cos\theta'_d}{1 + e_d \cos\theta'_d}\right) - \frac{\pi}{2}}{\sqrt{1 - e_d{}^2}} - \frac{e_d \sin\theta'_d}{1 + e_d \cos\theta'_d} \right] \tag{A12}$$

and the total time elapsed from the reference point (apogee) is

$$t_s = t_e + t_d \tag{A13}$$

where t_e is obtained from equation (A5).

The location of the entry point with respect to the apogee of the initial orbit is

$$\theta'_s = \theta'_e - \theta'_d \tag{A14}$$

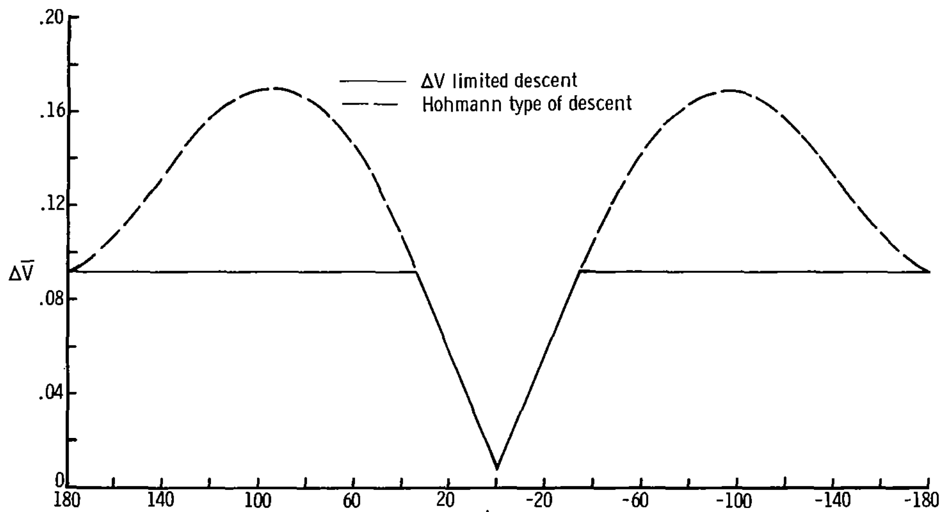
As an example of the analysis, consider an elliptical orbit with a 2-hour period. The orbit parameters are:

$$r_{a,e} = 5165 \text{ nautical miles}$$

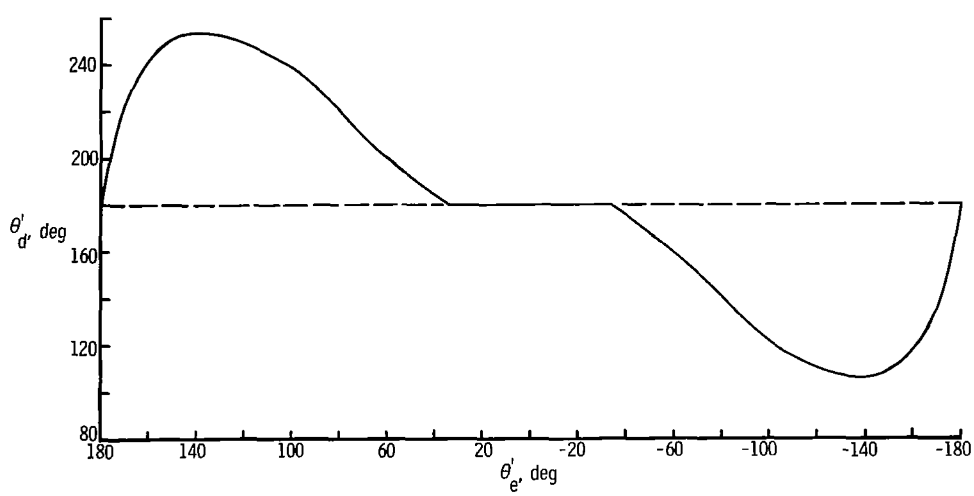
$$r_{p,e} = 3600 \text{ nautical miles}$$

$$e_e = 0.1793$$

Figure A2 summarizes the results obtained for return from this orbit. First, the $\Delta\overline{V}$ requirements for a Hohmann type of transfer from the elliptical orbit to the atmosphere were determined (dashed curve in fig. A2(a)). The maximum value of $\Delta\overline{V}$ obtained, 0.1693, would require that more than 40 percent of the spacecraft weight be allocated to the retropropulsion system for typical engine parameters ($I = 300$ sec, $\sigma = 0.1$). For this example, a maximum allowable value of $\Delta\overline{V}$ of 0.092 was selected. This value of ΔV requires that 24 percent of the total weight be allocated to the retropropulsion system. Note that 180° descent trajectories are possible only at perigee and

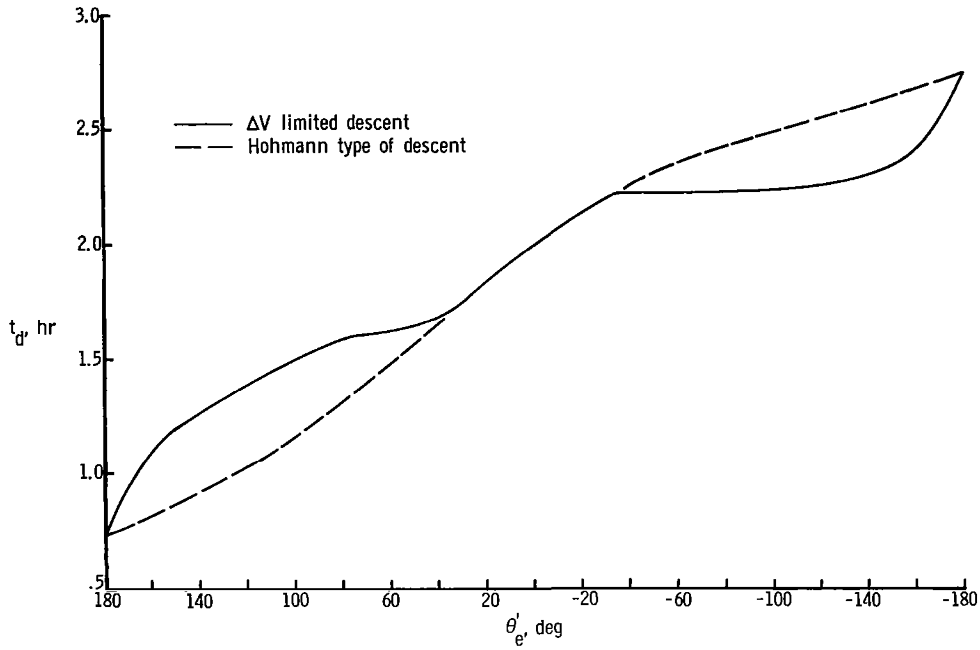


(a) Velocity increment.



(b) Longitudinal range angle for descent.

Figure A2.- Descent trajectory parameters for return from an elliptical orbit. $r_{a,e} = 5165$ n. mi.; $r_{p,e} = 3600$ n. mi.; $e_e = 0.1793$.



(c) Time from deorbit to entry.
Figure A2.- Concluded.

for values of θ'_e between +34° and -34°. The values of θ'_d required are shown in figure A2(b) with the larger values required when the spacecraft is moving from perigee to apogee. The associated descent times t_d are shown in figure A2(c).

The total angles traveled and times (referred to entry at zero time for deorbit at perigee) are presented in figure A3. As might be anticipated, the time required for the

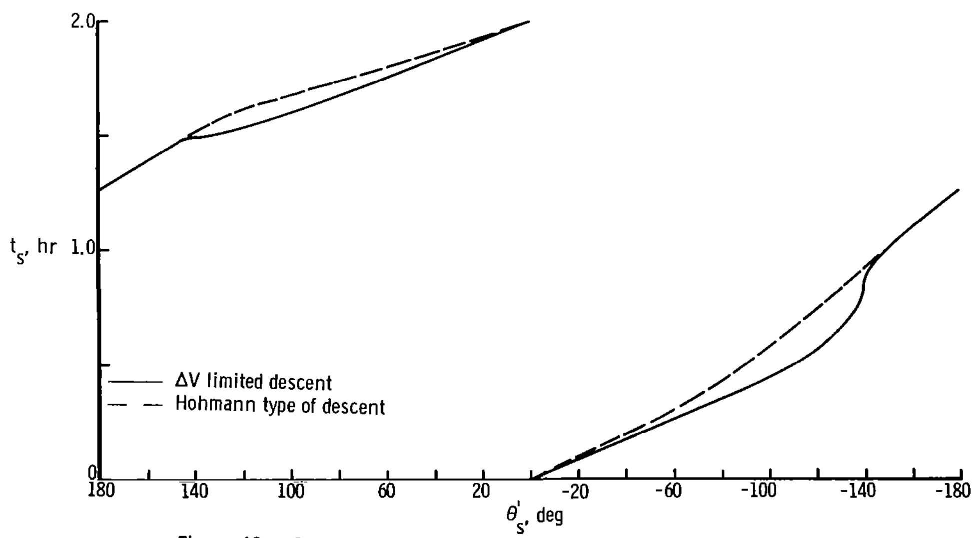


Figure A3.- Spacecraft motion for return from an elliptical orbit.

ΔV limited descents are greater than the 180° descent times for the spacecraft motion from perigee to apogee and less than the 180° descent times for the spacecraft motion from apogee to perigee.

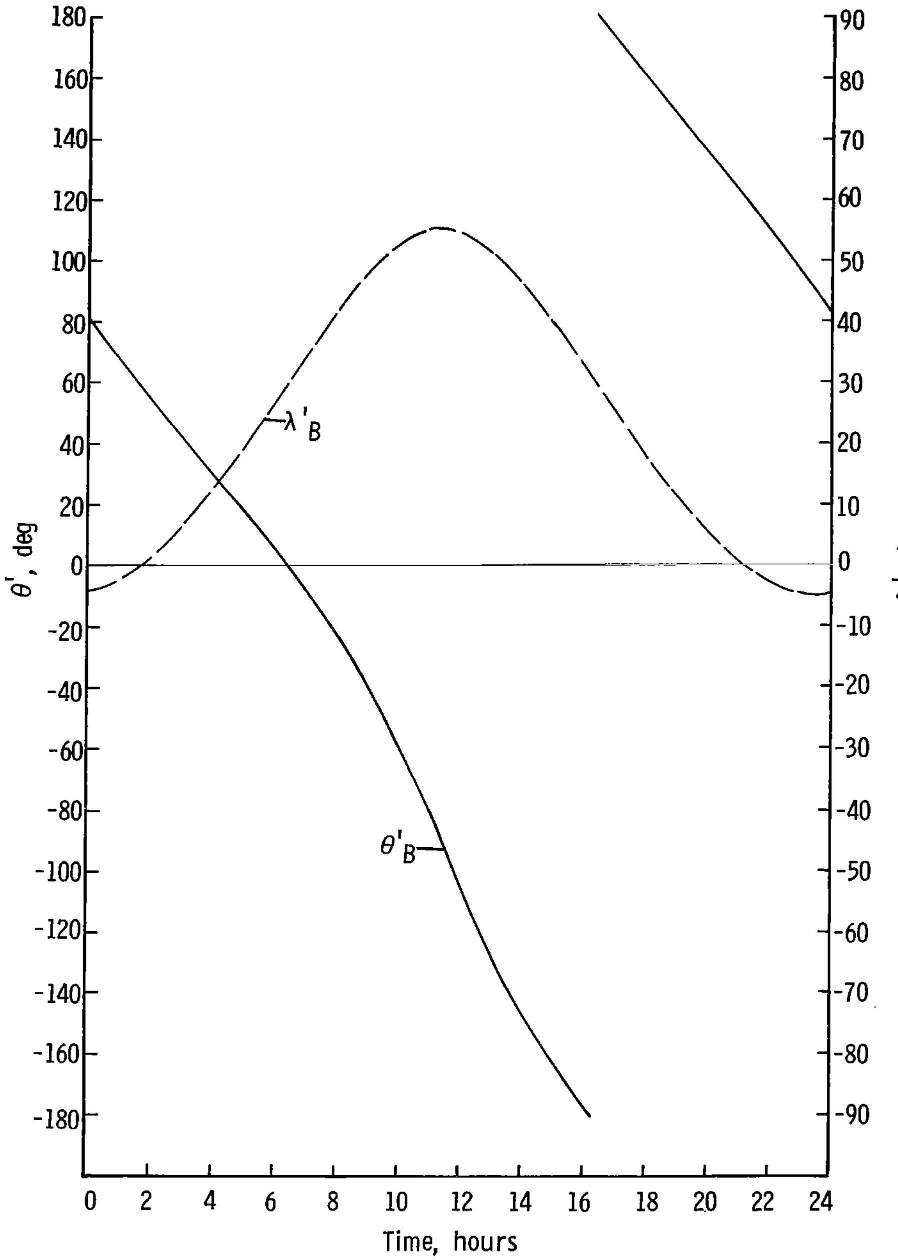
APPENDIX B

EXAMPLE PLOTS FOR COMPUTER-GRAPHICAL TECHNIQUE

Overlays 1 to 3, provided in the envelope in the back cover of this report, depict the motion of spacecraft on circular orbits with periods of 1.5 and 10 hours and on an elliptical orbit with a period of 2 hours. Return opportunities and lateral-range requirements may be defined for any vehicle for recall to any of the sites considered by placing the appropriate orbital overlay over any of figures B1 to B3. For circular orbits, any desired location of the spacecraft at zero time is obtained by moving the overlay horizontally. The return opportunities can then be defined as illustrated in figure 2. The intersection of θ'_S and θ'_B locates the time of the return opportunity during a given orbit. Reading vertically to the λ'_B curve yields the lateral-range capability required to reach the recovery site.

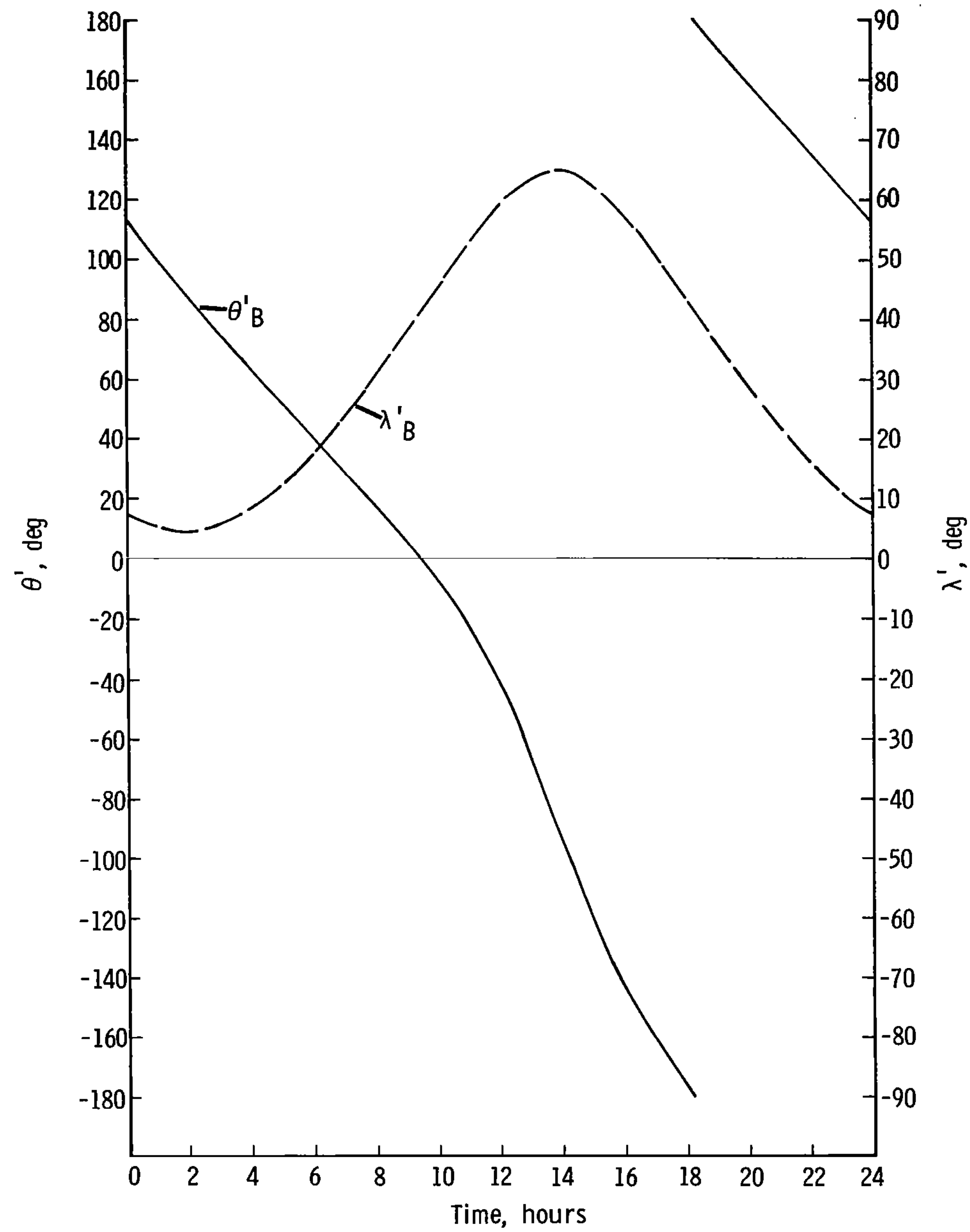
For elliptical orbits, the desired location of the spacecraft at zero time is obtained by moving the overlay both horizontally and vertically. The ordinate scale of the elliptical overlay indicates that the intersection of the orbital and equatorial planes ($\theta' = 0$) lies along the semimajor axis of the ellipse. If, however, the intersection of the orbital and equatorial planes does not occur at this point, the overlay must be shifted vertically to orient the overlay correctly (i.e., if $\theta'_S = 0^O$ at the semiminor axis, the overlay must be shifted by 90^O). Shifting the overlay horizontally then gives the desired value of θ'_S at $t = 0$.

Once the overlay has been properly oriented, return opportunities for elliptical orbits are obtained in the same manner as those for circular orbits.



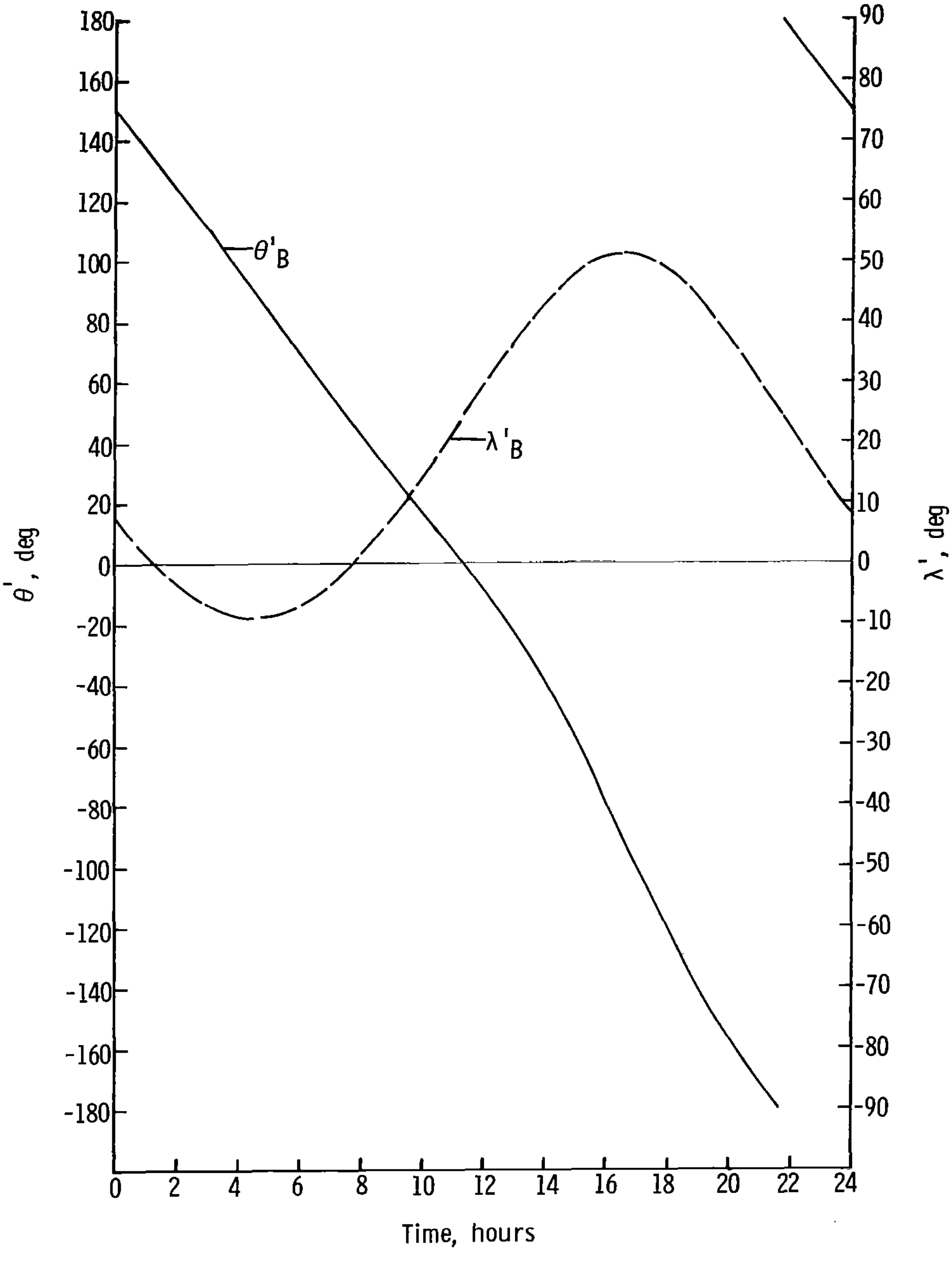
(a) Recovery site at Homestead AFB, Florida.

Figure B1.- Return from an orbit inclined -30°.



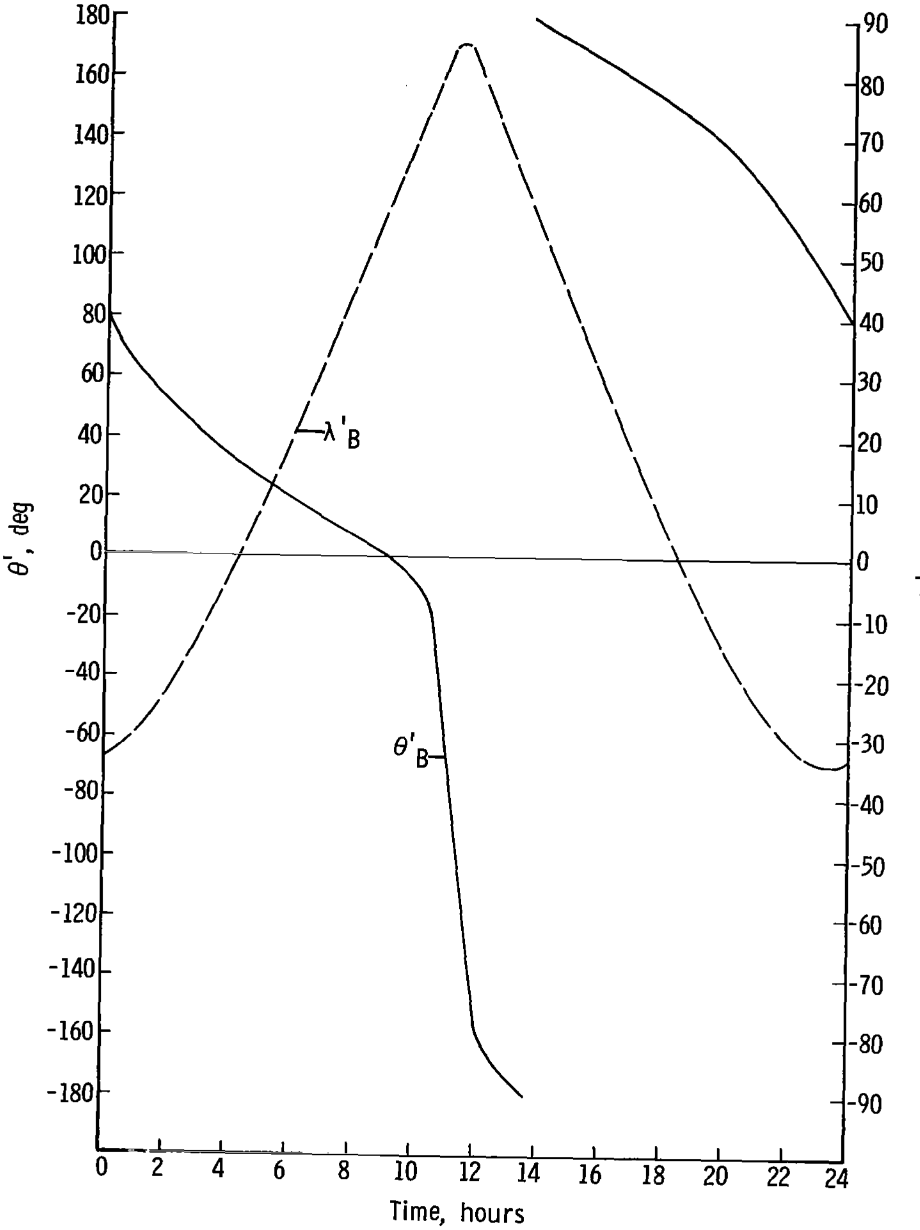
(b) Recovery site at Edwards AFB, California.

Figure B1.- Continued.



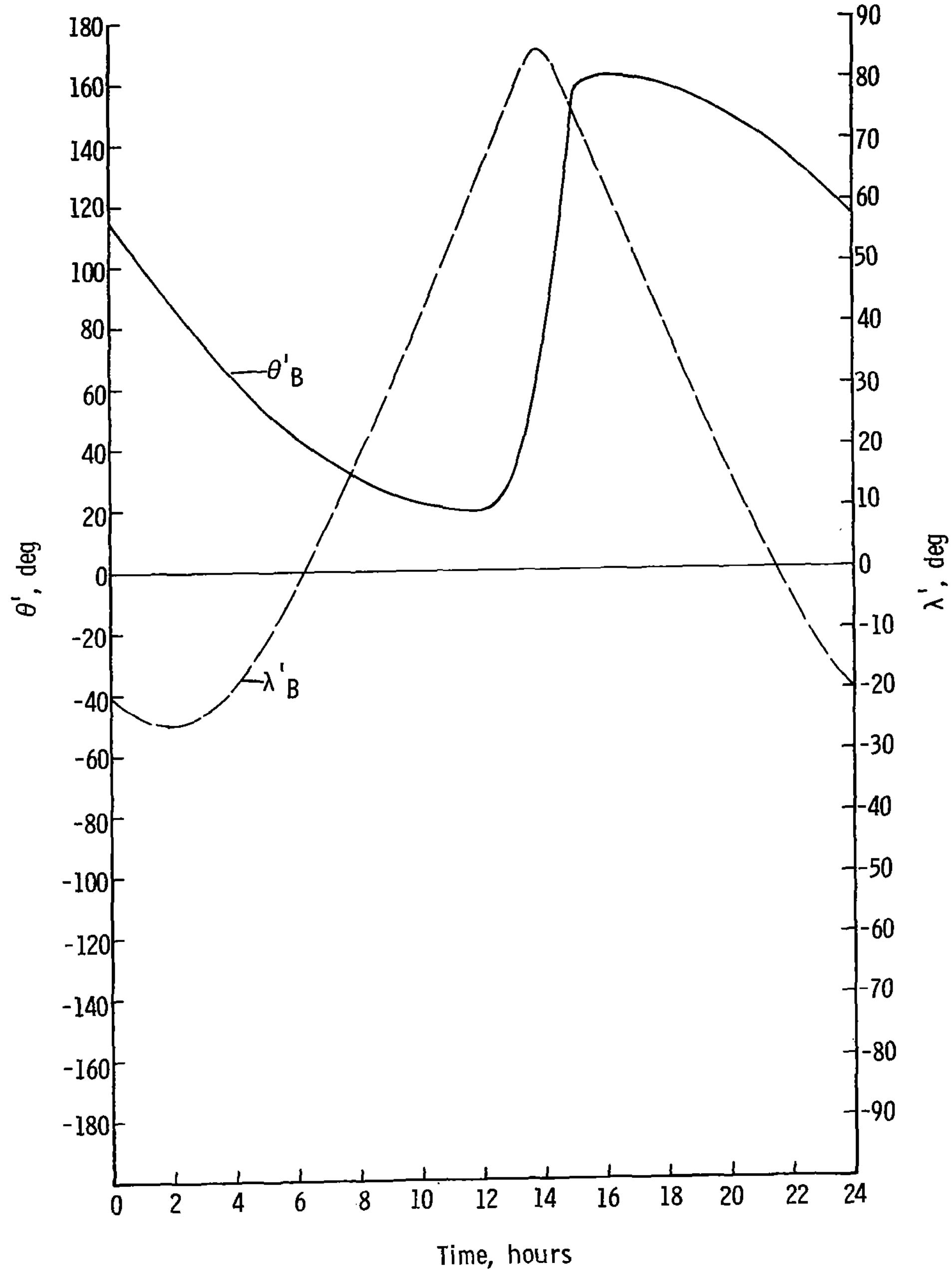
(c) Recovery site at Hickam AFB, Hawaii.

Figure B1.- Concluded.



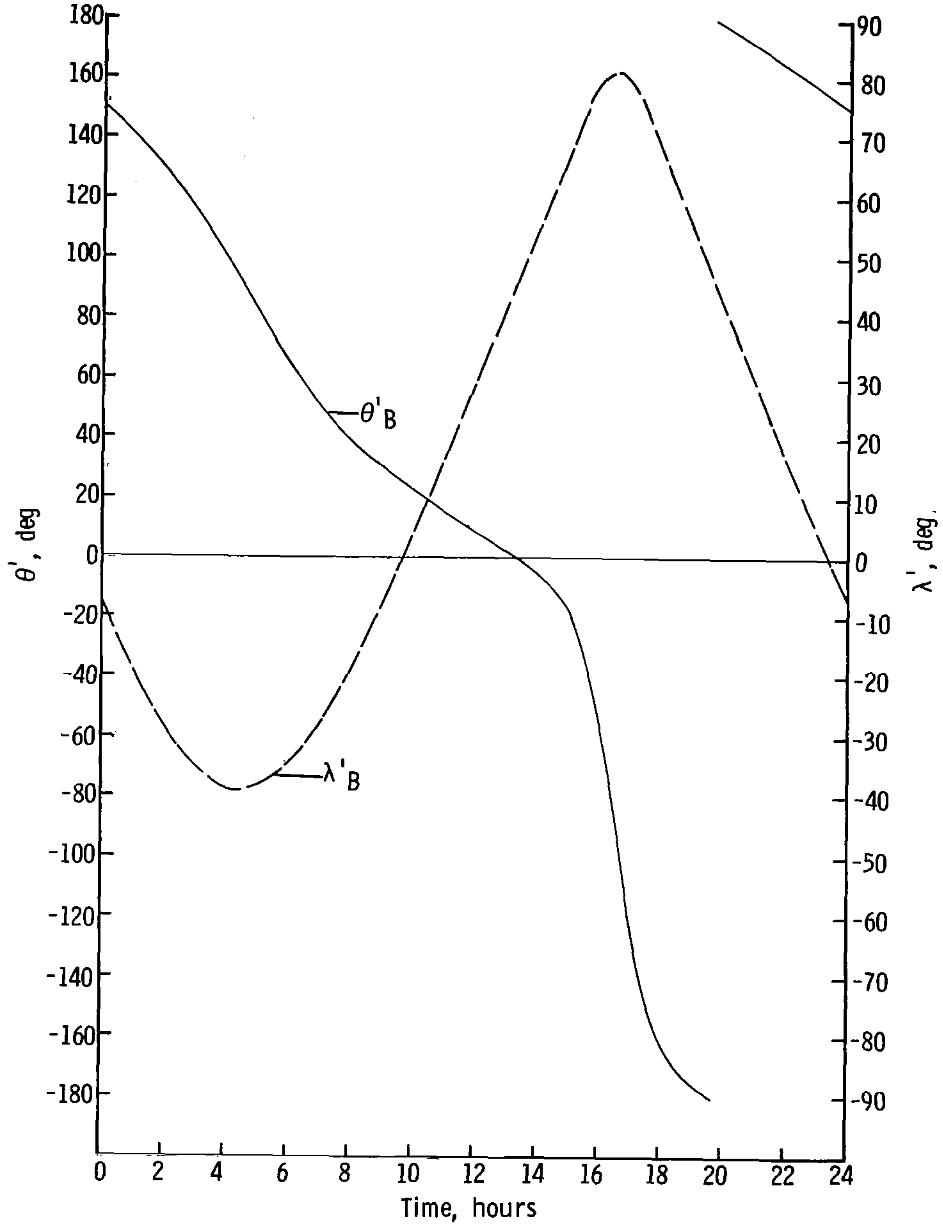
(a) Recovery site at Homestead AFB, Florida.

Figure B2.- Return from an orbit inclined -60°.



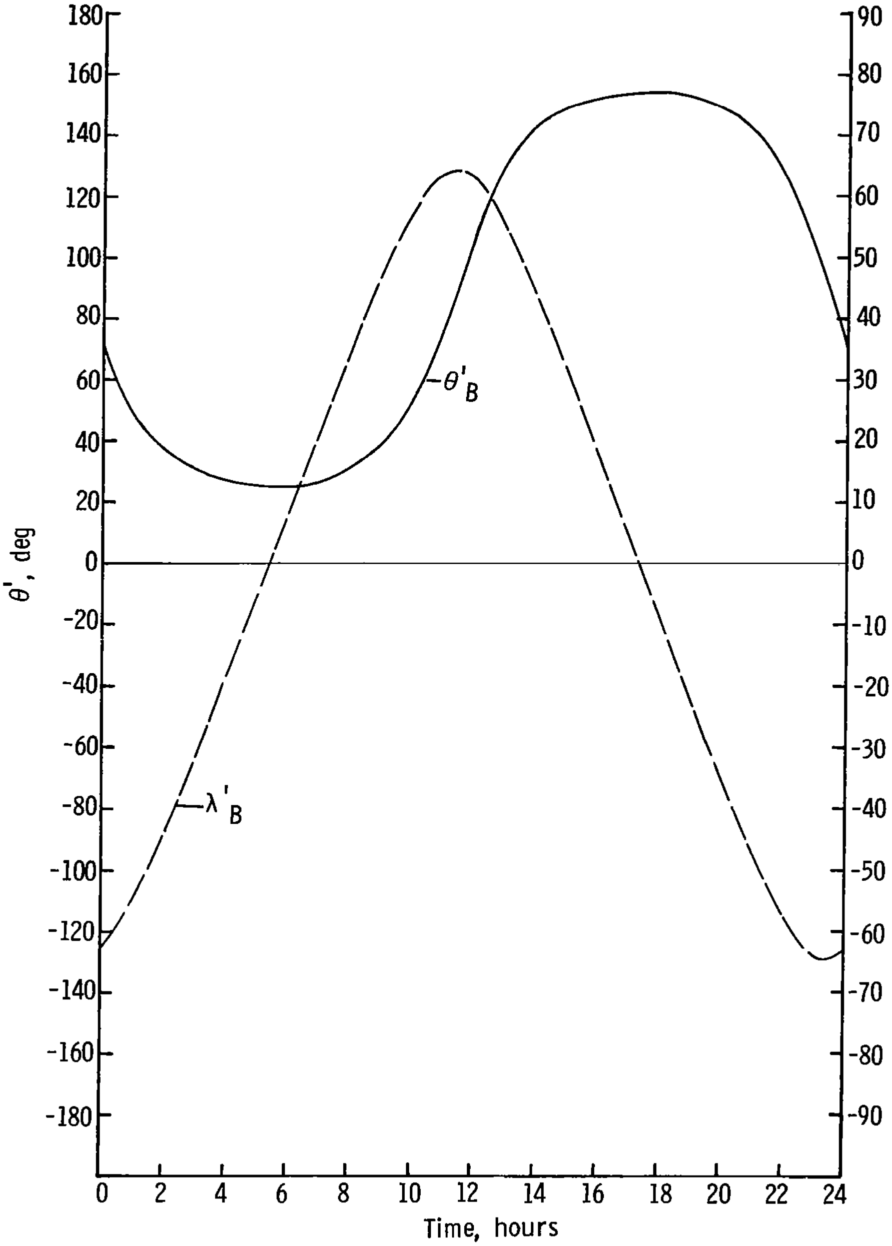
(b) Recovery site at Edwards AFB, California.

Figure B2.- Continued.



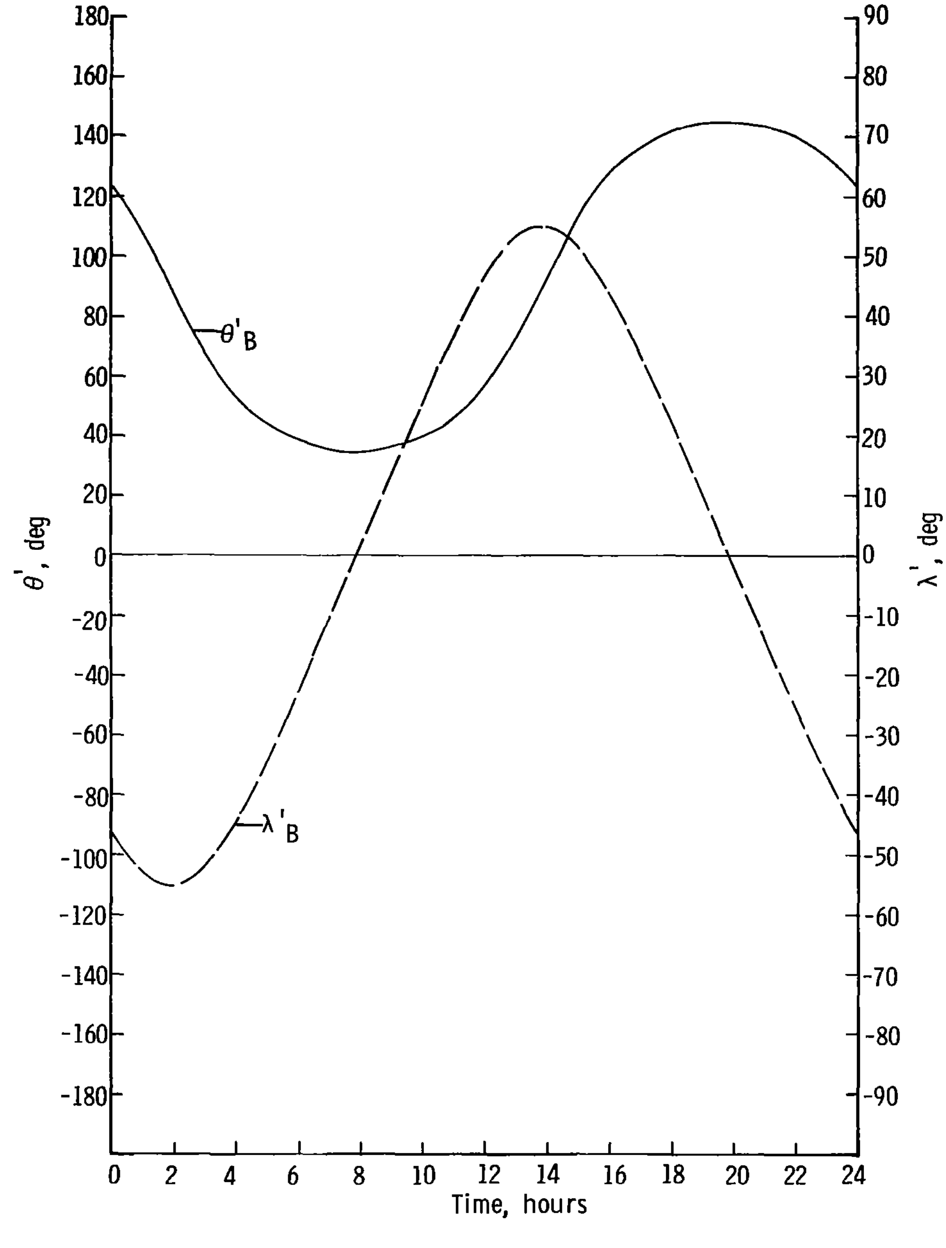
(c) Recovery site at Hickam AFB, Hawaii.

Figure B2.- Concluded.



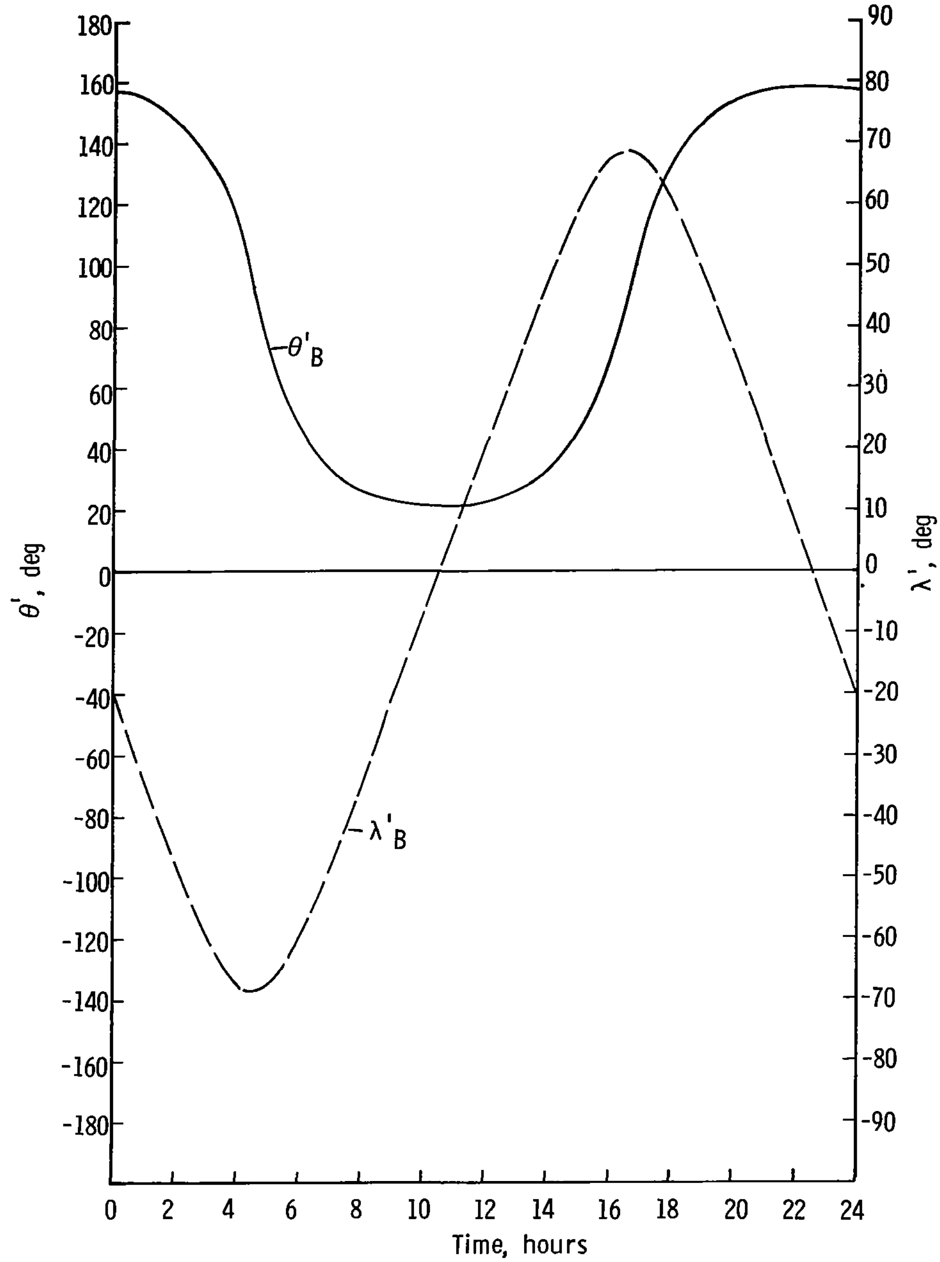
(a) Recovery site at Homestead AFB, Florida.

Figure B3.- Return from an orbit inclined -90° (polar).



(b) Recovery site at Edwards AFB, California.

Figure B3.- Continued.



(c) Recovery site at Hickam AFB, Hawaii.

Figure B3.- Concluded.

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